Waves & Coffee – The Hertzian Dipole antenna.

What is it all about & should we care about it?
By Stephen W2WF

Continuing what we saw in the last installment of this column, we will now look at a dipole antenna in the far field. Specifically we shall look at a very small dipole antenna from far away so that we can make some approximations in the math. This very small dipole antenna, relative to the wavelength of the transmitted wave, has historical significance since it was exactly what Heinrich Hertz studied back in 1888. In his honor it is called the "Hertzian Dipole" antenna.

Note that in 1888 when Hertz was studying this in Karlsruhe, Germany, maxwell's equations had only been discovered 23 years earlier. Also, note that Guglielmo Marconi would not do his first transatlantic transmission from Cornwall, England to Newfoundland, Canada for another 13 years. So Hertz was doing cutting edge experimental physics at that time.

It's particularly interesting to note what Hertz is quoted as saying of his first transmission of radio waves:

"It's of no use whatsoever, this is just an experiment that proves Maestro Maxwell was right - we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there."

So, let's look at the physics that we now understand were at play for Mr. Hertz' very small dipole antenna. Note that the math in this article may get to be a bit involved, so if you're keen on a challenge, let's plow ahead!

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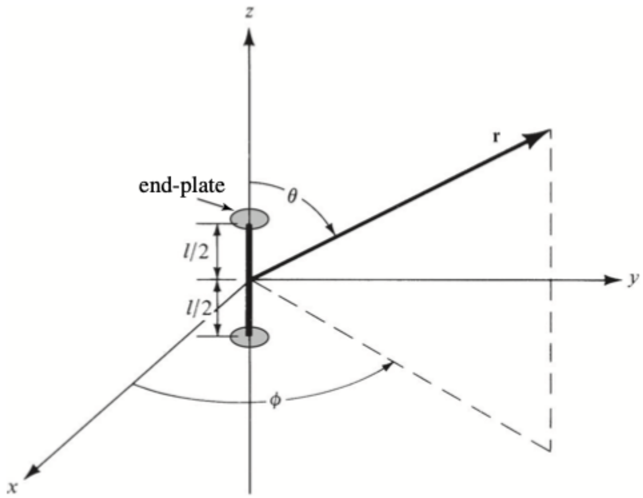
We'd seen how the quantity called the "Vector potential" $\boldsymbol{A}$ was our starting point to get to the Magnetic & Electric fields. We mentioned that our approach was going to be to go from $\boldsymbol{A} \rightarrow \boldsymbol{B} \rightarrow \boldsymbol{E}$

Let's start by finding the Magnetic field ***B*** from the Vector potential ***A***.



Recall from last time and using $j = \sqrt{-1}$, we had approximated our expression for ***A*** in the far field to be:

$$\boldsymbol{A}(x,y,z) = \frac{\mu_o}{4\pi}\frac{e^{-jkr}}{r} \int \boldsymbol{I_e}(x',y',z')dl'$$

For the Hertzian dipole, we consider the simplest current source $\boldsymbol{I_e}(x',y',z')$, namely $\boldsymbol{I_e}(x',y',z') = \boldsymbol{a}_z I_o$. In other words, we assume that the current is constant from top to bottom in our little dipole antenna. I say little since we're looking at it from the Far Field, in other words a large distance away from it. So it looks small in size from our perspective.

Anyway, since I_o is a constant, we can pull it out of the integral like this,

$$\boldsymbol{A}(x,y,z) = \frac{\mu_o}{4\pi}\frac{e^{-jkr}}{r} \boldsymbol{a}_z I_o \int_{-l/2}^{+l/2} dz'$$

Which we readily integrate to get,

$$\boldsymbol{A}(x,y,z) = \frac{\mu_o}{4\pi}\frac{e^{-jkr}}{r} I_o l\, \boldsymbol{a}_z$$

We should work in spherical coordinates (r, θ, φ) instead of (x, y, z) due to the symmetry of the problem. So we transform to spherical coordinates using the standard transformations that we look up in a math reference book,

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

and our expression for the vector potential ***A*** becomes:

$A_r = A_z\cos(\theta) = \frac{\mu_o}{4\pi}\frac{e^{-jkr}}{r} I_o l\cos(\theta)$

$A_\theta = -A_z \sin(\theta) = -\frac{\mu_o}{4\pi}\frac{e^{-jkr}}{r} I_o l \sin(\theta)$

$A_\varphi = 0$

Now that we know ***A***, we can get the magnetic flux ***B*** from the following definition:

$$\boldsymbol{B}(r,\theta,\varphi) = \nabla \times \boldsymbol{A}(r,\theta,\varphi)$$

Since the A_r and A_θ components don't have any φ dependence and since $A_\varphi = 0$, only one component of ***B*** is non-zero, the $\boldsymbol{a}_\varphi$ component:

$$\boldsymbol{B}(r,\theta,\varphi) = \nabla \times \boldsymbol{A}(r,\theta,\varphi) = \boldsymbol{a}_\varphi \frac{1}{r}\left[\frac{d}{dr}(rA_\theta) - \frac{d}{d\theta}A_r\right]$$

Since we know what A_θ and A_r are from the previous step, we do the derivatives and get,

$$\boldsymbol{B}(r,\theta,\varphi) = \boldsymbol{a}_\varphi \frac{jk\mu_o I_o l \sin(\theta)}{4\pi r}\left[1 + \frac{1}{jkr}\right] e^{-jkr}$$

Lastly, since we're looking at the *"Far field"* where *r* is very large, the second term in the brackets goes to zero and our B field equation finally becomes:

$$\boldsymbol{B}(r,\theta,\varphi) \cong \boldsymbol{a}_\varphi \frac{jk\mu_o I_o l \sin(\theta)}{4\pi r} e^{-jkr}$$

Once we have ***B*** we get ***E*** by multiplying times the intrinsic impedance Z_o and dividing by μ_o. This since the value of the intrinsic impedance in free space is $Z_o = \sqrt{\frac{\mu_o}{\epsilon_o}}$ or 377 ohms. We also note that the direction of the ***E*** vector will be perpendicular to both r and φ, so it will point in the θ direction:

$$\boldsymbol{E}(r,\theta,\varphi) \cong \boldsymbol{a}_\theta \frac{jZ_o k I_o l \sin(\theta)}{4\pi r} e^{-jkr}$$

The fields are then the real parts of these two final equations. That concludes the derivation of ***B*** and ***E*** for the Hertzian dipole antenna.

So, we see that using approximations for the far field and for the dimensions of the dipole antenna we can get the familiar doughnut lobes for our radiation pattern. This gives us insight that is applicable to the future study of other cases such as the small dipole and the half wave dipole antennas.

References:

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